

RESEARCH SUMMARY:

International Baccalaureate mathematics comparability study: Curriculum and assessment comparison

Summary developed by the IB Research department based on reports prepared by:

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With special thanks to Liz Bergeron, PhD (University of Wisconsin-La Crosse) for preparing a summary of the two full reports.



BACKGROUND

This study reviews the International Baccalaureate's (IB) Diploma Programme (DP) mathematics curriculums in comparison with the mathematics curriculums offered in a range of widely recognized education systems around the world. The four mathematics courses offered in the Diploma Programme—mathematical studies standard level (SL), mathematics SL, mathematics higher level (HL) and further mathematics HL—are compared with five mathematics qualifications chosen on the basis of their market share, the extent of their university recognition, geographical spread and their performance on PISA tests (where relevant).¹

1. Alberta Diploma (Canada): Alberta Mathematics 30-1, Alberta Mathematics 30-2, Alberta Mathematics 31
2. Advanced Placement (AP) (US and Canada, growing overseas market): AP Calculus AB, AP Calculus BC
3. GCE A levels (England and overseas market): A level Mathematics and A level Further Mathematics
4. Singapore-Cambridge GCE A Levels (SIPCAL) (Singapore): Singapore H1 Mathematics, Singapore H2 Mathematics, Singapore H3 Mathematics
5. Gāokǎo (China): Gāokǎo

This study is composed of two distinct components: a curriculum comparison and an assessment comparison.

RESEARCH DESIGN

To encourage an objective analysis, two independent researchers were commissioned to carry out the curriculum and assessment comparisons. The curriculum comparison was

performed by Dr Adriana Alcántara. Data was gathered, mainly from primary sources, consisting of the most recent guidance documents and course specifications or guides provided by the administering authorities of each qualification. Secondary sources were also used to enrich the data, namely existing international studies that have compared various mathematics qualifications. In order to establish an external framework for determining equivalencies between the qualifications, the textbooks and course outlines used by top global universities in science and engineering were referred to for defining objective categories of mathematics content.

The assessment comparison was carried out by UK NARIC, the UK's designated agency responsible for providing information, advice and opinion on vocational, academic and professional skills and qualifications from all over the world. A qualitative comparison approach was utilized. The awarding bodies' or government websites were reviewed and any publically available documentation was collected. The main documents searched for included: examination papers and markschemes, student or teacher handbooks/guides and grade distribution data.

FINDINGS

Overview of curricular similarities and differences

Similarities and differences in structure, content, cognitive demand, philosophical underpinnings and university recognition for the qualifications are discussed on the next page.

¹The focus of this study is on the qualifications themselves, not on the national or state systems that sustain them (in the relevant cases).

Structure

Three of the six qualifications analysed are bacalaureate-style qualifications (the Alberta Diploma, Singapore-Cambridge A Level curriculum and the DP) whose purpose is to provide students with a holistic education that covers a variety of subjects. The Chinese Gāokāo is a comprehensive examination that defines the content of the senior secondary curriculum; thus, it also requires students to follow a certain fixed combination of subjects. The AP and A levels, in contrast, provide stand-alone qualifications in a variety of subjects that can be taken independently of each other. Mathematics is a required component of only three of these qualifications: the Alberta Diploma, the Gāokāo and the DP.

Content

With the exception of the Gāokāo, all other qualifications offer several mathematics courses that correspond to different higher education pathways.

- **Require less mathematical preparation:** Alberta Mathematics 30-2, Alberta Mathematics 30-1 and IB mathematical studies SL are all meant to prepare students for university studies that require less mathematical experience. These are all “pre-calculus” courses that cover functions, trigonometry and geometry.
- **Require a solid base in mathematics:** A level Mathematics, Singapore H2 Mathematics and IB mathematics HL all cover calculus topics and prepare students for university studies requiring a solid base in mathematics. All three of these qualifications prepare students for entry into mathematics, engineering and science fields, and aim to develop students’ analytical skills in mathematics.
- **In-depth calculus that is equivalent to introductory university courses:** Alberta Mathematics 31, AP Calculus AB and AP Calculus BC focus on calculus and its applications. The purpose of all three of these courses is to provide students with an in-depth calculus course that is equivalent to a first semester or first year introductory university course on the subject.
- **Prepare students for university studies in highly mathematical disciplines:** Singapore H3 Mathematics, A level Further Mathematics, IB further mathematics HL and Gāokāo cover calculus and a range of advanced topics to prepare students for university studies in highly mathematical disciplines. All of these qualifications are aimed at preparing students for careers where mathematics will be essential.

Cognitive demand

The cognitive demand required for each course was analysed based on the course descriptions and available course materials in addition to examining the content. The table below lists the courses from lowest to highest level of cognitive demand.

Mathematics curriculum	Scope of content areas	Breadth of study
Alberta Mathematics 30-2	Does not cover trigonometry or calculus; in applied mathematics, only covers basic probability.	Covers the least number of topics in algebra and functions; does not require any other areas of pure mathematics; no coverage of applied mathematics except for one unit on probability.
Alberta Mathematics 30-1	Does not cover calculus but does cover two basic areas of trigonometry.	Like Alberta Mathematics 30-2, covers the least number of topics in algebra and functions; does not require any other areas of pure mathematics; no coverage of applied mathematics except for one unit on probability.
IB mathematical studies SL	Requires basic trigonometry topics (two out of five) and introduces differentiation in calculus (two out of 13 topics).	Covers a moderate number of topics in algebra and functions, some geometry; requires two other areas of pure mathematics and has good coverage of statistics topics (six out of eight).
Singapore H1 Mathematics	Does not cover trigonometry (expects some basic knowledge of this area as a prerequisite) but does cover six out of 13 calculus topics (including parametric functions).	Covers a moderate number of topics in algebra and functions; does not require any other areas of pure mathematics but has very complete coverage of statistics topics (seven out of eight).
IB mathematics SL	Covers four out of five areas of trigonometry and eight out of 13 calculus topics (including applications to kinematics).	Covers all topics in algebra and functions; requires two additional areas of pure mathematics and has good coverage of statistics topics (five out of eight).
A level Mathematics	Covers all topics in trigonometry and eight out of 13 calculus topics (including parametric functions and infinite sequences and series).	Covers all topics in algebra and functions; includes study of parametric equations and curves in geometry; requires three other areas of pure mathematics; has good coverage of statistics topics (six out of eight); also includes decision mathematics and mechanics.

IB mathematics HL	Covers all topics in trigonometry; covers 11 out of 13 calculus topics, including infinite sequences and series (two of these topics are optional, however; does not include parametric functions).	Covers all topics in algebra and functions; includes study of parametric equations and curves in geometry; requires five other areas of pure mathematics (one as a prerequisite) and makes one more optional; covers all statistics topics (eight out of eight, but four of these are optional); also provides discrete mathematics as an option.
Singapore H2 Mathematics	Does not cover trigonometry but expects all trigonometry topics to have been covered as a prerequisite; covers nine out of 13 calculus topics (including parametric functions and infinite sequences and series).	Covers all topics in algebra and functions (some as prerequisites); includes study of parametric equations and curves in geometry; requires four other areas of pure mathematics; has quite complete coverage of statistics topics (seven out of eight).
Singapore H3 Mathematics	Does not cover trigonometry but expects all trigonometry topics to have been covered as a prerequisite; covers 10 out of 13 calculus topics (including parametric functions and infinite sequences and series).	Covers all topics in algebra and functions (some as prerequisites); includes study of parametric equations and curves in geometry as a prerequisite; requires five other areas of pure mathematics (four as prerequisites from H2); does not require any applied mathematics topics.
A level Further Mathematics	Does not cover trigonometry but expects all trigonometry topics to have been covered as a prerequisite; covers nine out of 13 calculus topics (including parametric functions, infinite sequences and series, and hyperbolic functions).	Covers all topics in algebra and functions (most as prerequisites); includes study of parametric equations and curves, as well as polar coordinates, in geometry; requires six out of eight other areas of pure mathematics; requires all eight topics in statistics and provides two additional areas of applied mathematics as options—decision mathematics and mechanics.
IB further mathematics HL	Covers one new area in trigonometry—expects all other trigonometry topics to have been covered as a prerequisite; covers 12 out of 13 calculus topics (including applications to kinematics and infinite sequences and series).	Covers all topics in algebra and functions (all as prerequisites); includes study of parametric equations and curves in geometry; requires six out of eight other areas of pure mathematics; requires all eight topics in statistics; requires one additional area of applied mathematics—decision mathematics.

Although the cognitive demand of calculus courses were examined separately, AP Calculus BC was deemed to be the most cognitively demanding, followed by AP Calculus AB and Alberta Mathematics 31.

Philosophical underpinnings

All of the qualifications include “university preparation” as a part of the description. All of the qualifications except the Gāokāo describe the following learning goals for students. To:

1. enjoy mathematics
2. communicate mathematics
3. apply mathematics across contents and to the real world
4. incorporate technology with mathematics
5. build connections.

The Alberta Diploma and the DP are unique in their inclusion of international or global dimensions. The DP, the GCE A levels and the Singapore-Cambridge GCE A Levels all describe attributes of a learner they hope to develop—specifically, students who take responsibility and are self-directed, concerned citizens and confident.

Recognition at illustrative universities

At selected top global universities, DP, AP and A level qualifications have the greatest level of recognition overall. AP’s Calculus BC is the most widely accepted for advanced credit and AP courses overall seem to have the most detailed credit policies at all of the US-based universities examined. The other qualifications are recognized at a more local level but often do not have explicit policies formulated for them in these top-ranked higher education institutions.

Overview of similarities and differences in assessment among the curriculums

All six qualifications examined assess students through one or more external written examinations conducted under exam conditions. The most notable differences in assessment observed between the mathematics qualifications pertain to item type, marking approaches and grading systems. Table 2 displays each qualification with the item type utilized.

Table 1. Cognitive demand by course (from lowest to highest)



Qualification	Courses/ programme	Multiple choice	Short answer	Multi-part structured	Extended problem	Combination of multi-part structured and extended problem
DP	Mathematical studies SL		✓	✓		
	Mathematics SL		✓	✓	✓	✓
	Mathematics HL		✓	✓	✓	✓
	Further mathematics HL		✓	✓	✓	✓
Advanced Placement	Calculus AB	✓		✓		✓
	Calculus BC	✓		✓		
	Statistics	✓		✓		
GCE A level	A level Mathematics		✓	✓	✓	✓
	A level Further Mathe- matics		✓	✓	✓	✓
Gāokāo	Arts stream	✓	✓		✓	✓
	Science stream	✓	✓		✓	✓
Alberta Diploma	Mathematics 30-1	✓	✓			
	Mathematics 30-2	✓	✓			

Table 2. Comparative review of item type utilized

The second notable difference among qualifications is marking approaches. Across all mathematics within the DP and GCE A level, the markschemes provide a significant level of guidance to examiners on how to apply marking principles to incomplete answers or alternative methods. Students are expected to show their work; markschemes detail the number of marks associated with demonstrating an appropriate method. Alternatively, in Gāokāo and AP mathematics, a considerable proportion of the marks (50% in the case of the AP) are assigned to multiple-choice questions, where no marks are available for method, thereby placing more weight on providing an accurate answer in order to receive marks.

The final notable difference identified was grading schemes. DP mathematics courses allow for the most granular level of differentiation in student achievement by grade, which can be considered beneficial for facilitating identification of the highest levels of student achievement for competitive higher education admissions purposes. For the DP further mathematics HL, the GCE A level, the AP courses and the Gāokāo, the external assessment accounts for 100% of the student's final grade. The other DP mathematics courses all include internal assessment, contributing 20% to the final grade. Lastly, the Alberta Diploma also includes internal assessment, which constitutes 50% of the final grade.

SUMMARY

DP mathematics courses compare favourably with the other international qualifications examined in this study. The DP offers the greatest number of mathematical course options for students with different interests. Based on the criteria used for this analysis, the IB's further mathematics HL was determined to be the most cognitively demanding course among all of the curriculums examined, followed by A level Further Mathematics and Singapore H3 Mathematics respectively. With regard to assessment, the DP and GCE A level provide partial credit for accurate methods used, which is considered beneficial as it offers a more comprehensive record of student understanding. Lastly, the DP courses allow for the most detailed level of differentiation in student achievement by grade.

This summary was developed by the IB Research department. Copies of the full reports are available at ibo.org/en/about-the-ib/research/. For more information on this study or other IB research, please email research@ibo.org.

To cite the full reports, please use the following.

Alcántara, A. 2016. *International Baccalaureate mathematics comparability study: Curriculum and assessment comparison*. Bethesda, MD, USA. International Baccalaureate Organization.

The National Recognition Information Centre for the United Kingdom. 2015. *Assessment in upper secondary mathematics: A comparison between the International Baccalaureate Diploma Programme and international qualifications*. Bethesda, MD, USA. International Baccalaureate Organization.